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*John Mair and Domingo de Soto on the reduction of iterated modalities*
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ON THE REDUCTION OF ITERATED MODALITIES

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The two logicians considered in this paper lived a long time ago, and thus the reader may be surprised that they were even aware of what seems such a modern creature: the iterated modality. The iterated modality strikes one as a typically twentieth century sort of thing—a rather unlovely child of the modern logician's predilection promiscuously to engender strings of formulae. It is now well-known however that logicians were concerned with iterated modalities during the Middle Ages, and it will be the task of this paper to reveal that logicians of the early modern period proposed reduction rules for iterated modalities and to discuss their justifications for these rules.\(^1\) Thus, they attained

\(^1\) Klaus Jacobi finds references to iterated modalities in the work of William of Sherwood (fl. 1230-50) in (Jacobi, 1980), pp. 233-242, while Peter King, in the introduction to (Buridan, 1985), p. 81, claims that Buridan (fl. 1325-1358) seems to be using S5 because he used the characteristic reduction law of that system. A note of caution must be sounded concerning King's claim. It is not completely
a level of competence in this regard which had not been matched until the latter part of this century.

We begin in the first section by explaining what an iterated modality is and how various systems of modern modal logic contain equivalences which allow one to reduce the number of such modalities. We discover that the "strongest" modal system S₅ contains a theorem which we call the Doctrine of the Necessity of Modal Propositions. In the second section, we find John Mair espousing a limited version of this Doctrine. As we attempt to determine the cause for his commitment to the Doctrine², we turn in the third section to Domingo de Soto's presentation of Mair's view, which we learn is based on the concepts of term extension or "ampliation" and of second intentions. The fourth section will make use of our exposition of these latter two concepts to reconstruct Mair's reasoning and to explain the differences between his view and Soto's concerning the extension of the Doctrine. The concluding section provides a cursory examination of the philosophical value of their opinions.

I

When Hughes and Cresswell explain how the modal systems S₄ and S₅ differ from T, they point out that these systems contain reduction laws for the modalities where T has none. A reduction law is an equivalence "which enables us to replace some sequence of modal operators by a shorter sequence"³. Thus, for example, the formula

\[ Mp \leftrightarrow LMP \]

is a theorem of S₅, and implies that even though the formation rules of S₅ allow one to iterate the modal operators at least a potentially infinite number of times, there are only 6 distinct modalities in that system⁴. Thus, S₄ and S₅, in opposition to T which has no reduction laws, allow one to manipulate iterated modalities in a way that T does not. A modality is said to be iterated if and only if it contains two or more modal operators⁵.

The system S₅ in turn stands out from the others in that it is committed to the Doctrine of the Necessity of Modal Propositions⁶. Hughes and Cresswell describe the Doctrine in this way:

³. (Hughes, 1977), p. 44.
⁴. Ibid., p. 50.
⁵. Ibid., p. 47, where a modality is "any unbroken sequence of zero or more monadic operators (¬, L, M)".
⁶. We will refer to this, and nothing else, as "the Doctrine".
Some philosophers have maintained not merely that all necessary propositions are necessarily necessary, but that if a proposition possesses any modal characteristic (necessity, possibility, impossibility, contingency) it possesses that characteristic by necessity; others, however, have questioned or denied this. The unrestricted Doctrine of the necessity of modal propositions holds only in S5. Thus, it is true in S5 that

$$\text{mod}(\alpha) \rightarrow L\text{mod}(\alpha)$$

where 'mod($\alpha$)' means '$\alpha$' is "fully modalized." S4, of course, is not committed to the Doctrine since 'Mp $\rightarrow$ LMp' is not a theorem of S4.

The unnamed philosophers referred to by Hughes and Cresswell are presumably twentieth century figures. They probably have in mind twentieth century logicians wrangling over the virtues of one modal system as compared to all the rest.

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7. Ibid., p. 45.

8. $\alpha$ is fully modalized if and only if every propositional variable in $\alpha$ occurs within the scope of a modal operator. For a proof of the claim with regard to S5, see Ibid., pp. 127-129.

9. The Kneales, in (Kneale, 1962), pp. 550-568, provide a survey of twentieth century views on the subject. C. I. Lewis, Oskar Becker, and Von Wright all have had doubts about various reduction laws, while the Kneales, on the other hand "prove" the S5 formula 'Mp $\rightarrow$ LMp' on page 565.

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II

Perhaps the first philosopher to state a general reduction law for the modalities is John Mair (1469-1550). In 1508 he proposed a limited version of the Doctrine of the necessity of modal propositions. He claims that every true divided modal proposition is necessary, while every false one is impossible. A divided modal proposition, we are told, is a proposition in which the copula in accordance with itself or part of itself is determined by some one of the four modes: necessary, possible, contingent, or impossible. Such propositions are called "divided" because the mode mediates or divides the parts of the proposition. A composed modal proposition on the other hand "composes" the parts of the proposition and brings them together into one term.

Setting aside any problems surrounding the division of propositions into composed and divided propositions, we see that Mair's claim that every true divided modal proposition is necessary and every false one is impossible is a clear

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10. Wilhelm Risse reports and edition of the Introductoricum pertinere in Aristotelicam dialecticen appearing as early as 1508 in Paris in (Risses, 1965), p. 31. I have used the copy of the Paris, 1527 edition which can be found in the Bayerische Staatsbibliothek in Munich.


12. Duplex est modalis scilicet divisa quae est propositio cuius copula secundum se vel suam partem aliquo istorum quatuor modorum determinatur, & dicitur divisa quia modus mediat inter partes dicti & eas dividit. Alia est composita, quae ideo dicit corposita quia partes eius componuntur, & ad idem extremorum se applicant. Ibid., p. lxvi(verso).
restatement of our S5 thesis: \( \text{mod}(\alpha) \rightarrow L\text{mod}(\alpha) \) where \( \alpha \) is restricted to divided propositions. The second part of Mair's view is redundant since \( \neg \text{mod}(\alpha) \rightarrow L\neg \text{mod}(\alpha) \) is equivalent to \( \text{mod}(\alpha) \rightarrow L\text{mod}(\alpha) \), and \( \neg \text{mod}(\alpha) \rightarrow L\neg \text{mod}(\alpha) \) is a thesis of S5 but not of any weaker system\(^{13}\).

Unfortunately there are two puzzles about Mair's view. First, it is not clear from the text immediately surrounding his pronouncement of the Doctrine why he believes it is true. His assertion of the Doctrine appears in the midst of a discussion of truth conditions for divided de possibili propositions, and the only support he gives for the Doctrine are some examples. For example, he asks us to consider the proposition 'Socrates possibly runs'. This proposition is necessary, he continues, whether Socrates is running or not. Here Mair makes the interesting remark that

[Socrates] always capable of running (in potentia ad currendum), even assuming that his feet were removed\(^{14}\).

Apparently, Mair believes that potential beings "always" and necessarily exist. That Socrates is running is a contingent state of affairs, but that Socrates has the potentiality to run is necessary. Mair, however, never explains why this is so. Two further examples are offered. Since it is false that

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13. The proofs are left as exercises for the reader, but the latter claim at least should be clear if one notes that \( \neg Lp \rightarrow L \neg Lp \) is a substitution instance of \( \neg \text{mod}(\alpha) \rightarrow L \neg \text{mod}(\alpha) \) and that the former is equivalent to \( MLp \rightarrow Lp \), which in turn is equivalent to our version of the S5 axiom: \( Mp \rightarrow L Mp \).

14. Sortes possibiliter currit est necessaria sive enim Sortes currat sive non semper est in potentia ad currendum, dato etiam quod e pedes abscondatur. (Mair, 1527), p. lxvii(verso).

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Socrates is possibly a donkey, it is also impossible that he is possibly a donkey. But it is necessary that Socrates possibly does not run\(^{15}\).

After listing these examples, he attempts to extend the Doctrine to divided propositions de necessario and de impossibili by means of what is, I'm afraid, an implausible argument:

If one proposition is necessary, it's contradictory is impossible, and vice versa. Therefore [sc. divided] propositions de impossibili and de necessario turn out necessary or impossible from the given principles\(^{16}\).

The argument is implausible because it is either trivial or inadequate. On one reading, Mair may be telling us that if we assume the principle that divided propositions de necessario are necessarily necessary —the S4 principle \( Lp \rightarrow LLp \)—then this principle also holds true for impossible propositions. That is, if \( Lp \rightarrow LLp \) is a theorem, then so is \( L \neg p \rightarrow LL \neg p \). This latter derivation is not trivial in the sense that it is obvious to us with our knowledge of standard systems of modern modal logic hence there seems to be no reason for him to include it. On the contrary, if his aim were to point out this derivation, he has explained as clearly as anyone could in a restricted amount of space why it works. The contradictory of a necessary proposition \( p' \), namely \( \neg p' \), is certainly impossible. We accept this nowadays as second nature since we usually translate 'it is impossible that \( p' \)' into modal logic as \( L \neg p' \). It is trivial rather

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because it presupposes the very Doctrine we would like to see proven or at least explained.

One might instead read Mair as saying that the claim: "divided propositions de necessario (impossibili) are necessarily necessary (impossible)" follows from the claim that divided possible propositions are necessarily possible. If he does think this is true, then of course he would be correct and interestingly correct. His intuition in that case would be that \( Lp \rightarrow LLp \) follows from \( Mp \rightarrow LMp \), or to put it another way, the characteristic axiom of S4 is implied by the characteristic axiom of S5. Unfortunately the claim that "if one proposition is necessary, then its contradictory is impossible and vice versa" seems woefully inadequate as a proof of this implication, no matter how charitably one interprets it. Since Mair's apparent defense of the Doctrine of the necessity of divided modal propositions is quite weak, it remains at present a puzzle as to why he thinks it is true.

The second puzzle about Mair's stand on the Doctrine is that he limits it to divided modal propositions. Why is it that he does not extend the Doctrine to composed modal propositions? We will try to answer this latter puzzle first since its solution points to a solution of the first, and we will find the answer to it in the work of Mair's critic, the Spanish logician Soto.

III

Domingo de Soto (1495-1560) accepts the Doctrine of the necessity of modal propositions unqualifiedly:

Every composite modal proposition is either impossible or necessary just like the divided ones.\(^{17}\)

Soto asserts this in the midst of an argument against certain logicians to whom he refers only as "Summuliets". These logicians hold the view, he says, that the modal expressions in composed modal propositions stand (supponit) as second intentions for propositions, while the modal expression in divided modal propositions refer as first intentions.\(^{18}\) This view, he says, implies that the modal expression in a composed modal proposition does not extend (ampliat) the denotations or suppositions of its terms, although it does extend them in divided modal propositions.\(^{19}\) This lack of term extension or "ampliation" in composed modal sentences is thought to imply, Soto continues, that composed modal propositions are contingents.\(^{20}\)

\(^{17}\) Omnis modalis composita vel est impossibilis vel necessaria sicut divisa. (Soto, 1980), p. 74 (verso).

\(^{18}\) At dubium est circa has differentias [sc. inter modalem compositionem & divisam] utrum modus in modali composita supponat secundae intentionaliter pro propositione. Nam ita fere Summuliets. Constituunt enim secundam differentiam, quia modus in composita stat secundae intentionaliter pro propositione, & in divisa primae intentionaliter. \(ibid.\), p. 74 (recto).

\(^{19}\) Nam sequitur ex opinione istorum ut ipsi concedunt quia modus in modali composita non ampliat, licet ampliat in divisa. \(ibid.\).

\(^{20}\) Ex quo inferunt secundo quod modalis composita est contingens. \(ibid.\). Mair does not think that all composed modal propositions
Soto thus introduces two notions to explain why the "Summulists" think that the Doctrine does not hold in the case of composed modal propositions: first, that of terms of "second intention", and second, that of extension or "ampliation" of terms. Mair himself uses these notions, and in fact he holds most of the views which Soto ascribes to the "Summulists".

With regard to second intentions, Mair tells us that the modal expressions are to be understood in two ways:

Is one way [modal expressions are understood] as second intentions and in that case 'possible' stands for a possible proposition and 'necessary' for a necessary proposition. In another way they are understood as first intentions, and so in two ways. In the first they are understood syncategorically, and these determine the copula...In another way they are understood categorically, and then 'necessary' means exactly the same as 'necessary being' 21.

The modal expression 'possible' in the proposition 'that Socrates runs is possible' 22 is a second intention. This are contingent. He gives at least on example of an impossible one on p. lxvi (verso) of (Mair, 1527): "Stans possibiliter est sedens, vel capitur in sensu composito, & sic ipsa est false naturaliter loquendo & impossibilis, in sensu diviso ipsa est necessaria ...".


22. I use the form 'p is possible' instead of 'it is possible that p' because the former reveals more clearly that Mair sees composed modal propositions to have the form 'A is B' and 'possible' is viewed as a term in propositions of that form. In Latin of course this is clear, and it is also clear, as Mair himself says, that the modal expression can stand in both the subject and the predicate place in composed modal sentences: "Non refert sive modus in fronte sive in terto pontur. Utraque istarum est modalis composita, Socratem curre est possibile, contingens est antichristum forse" (Mair, 1527), p. lxvi (verso).

In English, however, this would fail on grammatical grounds because 'possible is (that) Socrates runs' is ungrammatical.

23. For more careful and accurate presentation of Mair's view concerning second intentions see (Hickman, 1980), pp. 84-91 and (Broadie, 1985), p. 42.

this time\textsuperscript{25}, that Mair would make them the referents of 'possible' in such propositions. However, at first glance, the following passage seems to argue against this interpretation:

If any proposition is contingent or possible, then that [composed modal proposition] second intentionally is [also contingent or possible]. If 'Socrates runs' did not exist, then this is false: 'that Socrates runs is possible'\textsuperscript{26}.

Specifically the phrase \textit{in rerum natura} in this passage suggests that the proposition 'Socrates runs' must exist not merely as a mental entity but in some extramental fashion. Thus, Mair could be understood as endorsing the view that 'possible' in such propositions refers only to actual spoken or written propositions.

Although this is a possible interpretation, Soto seems to rule it out by including mental propositions as acceptable referents of 'possible' in composed \textit{de possibilibi} propositions when he presents Mair's view\textsuperscript{27}. One of his objections to Mair's view concerning composed modal propositions (which he believes is "fabricated without reason") is this:

Even though there were no proposition except this: 'that Peter disputes is possible', it is true and necessary. Indeed before the

\textsuperscript{25} Mental propositions, for example, are usually viewed as the bearers of truth by the school. See (Nuchelmans, 1980), pp. 10 and 116-7.

\textsuperscript{26} Si aliqua propositio est contingens vel possibilis, secunde intentionaliter ipsa est. Si nulla sit in rerum natura 'Sortes est', ista est falsa, 'Sortem currere est possibile'. (Mair, 1527), p. lxvii verso.

\textsuperscript{27} Our conclusion (see below) that Mair's phrase 'possible propositions' means "possibly true actual propositions" supports this view that 'possible' may refer to mental propositions.

creation of the world when there were no propositions, such was the case as is now signified by it\textsuperscript{28}.

Before the creation of the world, it was generally thought that there would be no propositions of \textit{any} kind\textsuperscript{29}. Therefore one is compelled to admit that the phrase \textit{in rerum natura} can be said of the contents of human minds.

The second notion Soto and Mair use to explain Mair's view that composed modal propositions are not necessary is that of term extension or "ampliation". According to Mair:

Composed modal propositions further differ from divided in such a way that the modal expression making a composed modal proposition does not extend ... but the modal expression making a divided modal proposition extends unless restricted\textsuperscript{30}.

Ampliation or term extension is a form of supposition. For the purpose of our discussion, it is enough to understand the supposition of a term as that entity or entities to which that term refers. Usually the supposition of a term is restricted to that entity or entities existing at the time ex-

\textsuperscript{28} Sed haec omnia sunt absurda, & sine ratione conficta. Manifestum enim est modum perinde ampliare in composita sicut in divisa. Et quanquam nulla sit propositio praeter hanc, Petrum disputatione est possibile, haec est vera & necessaria. Imo ante creationem mundi quando nulla erat propositio ita erat sicut nunc per illam significatur. (Soto, 1980), p. 74(verso).

\textsuperscript{29} Soto does not seem to subscribe to the opinion that, when all else fails, make God a proposition, although Ashworth tells us that some logicians of the time did. (Ashworth, 1978), pp. 104-5, where she tells us: "John Mair was content to say that there was no truth before there were any propositions to bear it'.

\textsuperscript{30} Caeceterum modalis composita a divisa sic discrepat, nam modus faciens modalem composam non ampliat ... at modus faciens modalem divisam ampliat nisi restringatur. (Mair, 1527), p. lxvi(verso).
pressed by the tense of the verb in the proposition within which that term appears. However, in medieval and renaissance logic, it was accepted that the supposition of a term could be extended to other times and other modalities by variation in the tense of the verb or through the addition of terms (like modal expressions) to the proposition. There were five kinds of term extension or ampliation: extension to things past, present, future, possible, and imagined\(^31\).

Mair’s claim that the modal term in composed modal propositions does not have ampliation means that the modal term only refers to, or takes as its supposition, those entities existing at the time indicated by the tense of the verb in the proposition in which the expression occurs. This result rules out one possible misunderstanding of Mair’s view that the expression ‘possible’ in composed propositions de possibili refers to “possible propositions”. The misunderstanding may arise since this latter phrase could be understood in two ways. On one hand, Mair may be claiming that the term ‘possible’ in composed sentences de possibili refers to propositions which do not actually exist, but only possibly exist. On the other, he may be claiming that an actual mental, spoken, or written proposition is possibly true. In this second case, he would be saying that there is an actual written, spoken, or mental proposition the terms of which stand for possible particulars.

Clearly the latter interpretation is best, and we find that Soto interprets him in this same way:

Therefore, in this proposition ‘that Peter disputes is possible’, ‘possible’ is taken to refer only to an actually existing proposition, because ‘possible’ as a second intention does not extend its


**IV**

We are now in a position to reconstruct Mair’s view concerning the nature of the modal term in a composed *de possibili* propositions, by stating the truth conditions of such sentences via figure 1.

‘That Socrates runs is possible’ is true if and only if ‘that Socrates runs’ and ‘possible’ stand for the same thing: in this case the actual mental, written, or spoken proposition ‘Socrates runs’. ‘Socrates runs’ is *possibly* true if and only if ‘Socrates’ and ‘running thing’ stand for (in the extended sense) the same possible entity. Since propositions are thought to be fleeting, momentary objects by logicians of this tradition\(^34\), there will be times at which the proposition ‘Socrates runs’ does not exist as, for example, before the creation of the world when there were no propositions. At such times, ‘that Socrates runs is possible’ will be false, and hence it is not necessary.

\(^{32}\) Itaque in hac propositione, Petrum disputare est possibile, ly, possibile, solum accipitur pro propositione existente in rerum natura, quia ly possibile, secundae intentionaliter no ampliat, quantum ad existentiam propositionis: sed quantum ad veritatem illius, iuxta ampliationem de ly possibile. (Soto, 1980), p. 74(recto).

\(^{33}\) In modalibus compositis alii termini a copula & modo te- dentur materialiter, non autem in divisa. (Mair, 1527), p. lxxvi(verso).

\(^{34}\) (Ashworth, 1978), pp. 83-84.
It is always difficult to determine exactly what a proposition signifies according to the logicians of this period. The main difficulty is that few if any of these logicians distinguished the sense of a proposition from its reference\(^{36}\). As a result one is usually given a theory of reference when one would like a theory of sense. Soto himself sets aside the question of what is signified by propositions in this context. He does not think that this is the place to dispute about what is signified by the proposition, what they, the Summulists, call the complexe significabile. It is enough for Summulists, he continues, that the dictum in a composed modal proposition and its corresponding de inesse proposition signify what is signified by the terms of such propositions\(^{37}\). So at least for the purpose of this discussion, we say that what is signified by 'man is animal' is "man who is animal"\(^{38}\).

The result can be represented by figure 2. All of the arrows in figure 2 represent extended or "ampliated" supposition of the terms.

The basic difference between Soto's and Mair's interpretation of the term 'possible' in composed modal propositions of this sort can now be succinctly stated. For

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\(^{36}\) (Ashworth, 1974), p. 47.

\(^{37}\) Hic non est locus disputandi de significato propositionis, quod vocant complexe significabile, videlicet quod res sit hominem esse lapidem, an sit homo vel lapis vel utriusque. Satis sit Summulistis propositionem & dictum infinitivi easdem res significare, quae significatur per extrema. (Soto, 1980), p. 74(verso).

\(^{38}\) Sed pro nunc dicimus quod non est alia res, sed hominem esse animal, est homo qui est animal. \textit{Ibid.}
Mair, 'possible' stands for an actual proposition while Soto believes it stands for a possible individual.\textsuperscript{39}

'Socrates possibly runs' is necessary if true because it refers to a possible man who is running. Further, it could not be the case that this proposition, or those of the same form, fail to refer to this possible man. So 'Socrates possibly runs' is necessarily true.

An answer to this question depends on how seriously they took this reference to possible individuals. There are at least two views concerning their seriousness in accepting the existence of possible individuals. On one view, "nominalist" logicians have no qualms about nonexistent possible objects as long as they are individuals. On another, they only accept actual, presently existing individuals, and reference to things which exist at other times or which possibly exist entails no ontological commitment to such things. Reference to things existing at other times than the present, or to possibilities is only a "manner of speaking".\textsuperscript{40}

In either case, both alternatives raise serious philosophical problems although the problems surrounding the first may be more intractable than those surrounding the second. An appeal to "ways of speaking" in order to provide a foundation for the Doctrine is very unconvincing since there are consistent "ways of speaking", for example T, which are not committed to the Doctrine at all. Nor could one appeal to ordinary language to discover the best way to talk about modalities since our intuitions arising from our knowledge of one or more natural languages are notoriously weak concerning iterated modalities. The reification of possible individuals on the other hand is fraught with difficulties as we all know. In addition to Quinean objec-

\textsuperscript{39} Mair's analysis of divided de possibili propositions is similar to Soto's analysis of such composed propositions. The syncategorimatic term 'possibly' extends or "ampliates" the suppositions of the terms to include possible individuals. Thus, 'Socrates possibly runs' is true if and only if 'Socrates' and 'running man' refer to the same possible person. See above and (Mair, 1527), p.lxvii(verso).

\textsuperscript{40} Both interpretations have been offered by King in (Buridan, 1985). He suggests the first on p. 327, note 8, while he suggests the second on pp. 55-6.
tions concerning identity conditions for possible individuals, problems more likely to arise in the sixteenth century must be faced. Could God, for example, destroy the possible Socrates, thus changing the truth value of 'Socrates possibly runs'? Or is the possible Socrates an eternal, immutable object? Then is he (it?) a created object? If not, is the possible Socrates "absolutely necessary", a status usually reserved for God? We cannot explore these problems here but that the question of the truth of the Doctrine is even meaningful to sixteenth century logicians clearly reveals, at least, a sophistication in dealing with the problems surrounding modal logic which rivals our own.

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